

# MEMORANDUM

To: Stan Carpenter  
 From: William Keyes  
 Date: 13 September 1965  
 Subject: HYPERGEOMETRIC DISTRIBUTION:

## 1. EXPONENTIAL FAILURE LAW:

To approximate the actual failure distribution, the exponential failure law is assumed. Let F denote the number of components which would fail when N components of MTTF = M hours are tested for t hours each. Then

$$F = N \left( 1 - e^{-\frac{t}{M}} \right) \dots \dots \dots (1)$$

## 11. HYPERGEOMETRIC DISTRIBUTION:

Given a lot of N gyros of which F (determined from equa. (1)) are defective, what is the probability of finding i defectives in a random sample of n units?

Let X = hypergeometric random variable, the number of defectives drawn from a sample of n gyros, of which F are defective.  
*which were drawn from a population of N gyros, (11/9/66)*

$$P(X = i) = \frac{\text{No. of favorable events}}{\text{No. of possible events}} \\
= \frac{\text{No. of ways to get } i \text{ defectives and } (n-i) \text{ non-defectives}}{\text{No. of ways to select } n \text{ samples from lot size } N}$$

$$\text{Numerator} = (\# \text{ ways to select } i \text{ defectives from } F \text{ defectives}) \\
\times (\# \text{ ways to select } (n-i) \text{ non-defectives from original } (N-F) \text{ lot of non-defectives})$$

$$= \binom{F}{i} \binom{N-F}{n-i} \text{ where } \binom{F}{i} = \frac{F!}{i! (F-i)!} \text{ etc.}$$

Denominator = # Combinations of N objects taken n at a time.

$$= \binom{N}{n}$$

Let  $p(i; N, F, n)$  = probability of finding i defectives in a sample of n gyros drawn at random from a lot of N gyros of which F are defective.

$$\text{Then } p(i; N, F, n) = \frac{\binom{F}{i} \binom{N-F}{n-i}}{\binom{N}{n}} \dots \dots (2)$$

The sum of hypergeometric probabilities must be 1

$$\sum_{i=0}^N \frac{\binom{F}{i} \binom{N-F}{n-i}}{\binom{N}{n}} = 1$$

defining  $\binom{N}{n} = 0$  when  $n > N$

With an acceptance number of failures C, a partial sum determines confidence factor:

$$\sum_{i=0}^C$$

$p(i; N, F, n)$  = sum of probabilities of finding  
i = 0, 1, . . . . C defectives

Define confidence factor P

$$\sum_{i=0}^C p(i; N, F, n) = \sum_{i=0}^C \frac{\binom{F}{i} \binom{N-F}{n-i}}{\binom{N}{n}} \equiv 1 - P \dots \dots (3)$$



To establish an acceptance test which will project a MTTF of  $M = r t$  (  $t$  = testing time ) with confidence  $P$  from a lot of  $N$  units with acceptance number  $C$ , the number of samples,  $n$ , which are needed is found:

- a. Calculate  $F$  from eq. (1) to nearest integer
- b. Calculate  $n$  from eq. (3)

If  $n$  units from a lot of  $N$  gyros are tested for  $t$  hours each and  $C$  random failures are observed, the MTTF with confidence  $P$  is calculated:

- a. Solve eq. (3) for  $F$
- b. Solve eq. (1) for  $M$

#### EXAMPLE:

In a procurement of 100 units, the MTTF is required to be 100,000 hours with 80% confidence and acceptance no. of failures  $C = 1$ . Find minimum sample size for 10,000 hours of testing.

$$\underline{a.} \quad F = N (1 - e^{-\frac{t}{M}}) = 100 (1 - e^{-0.1}) = 10$$

$$\underline{b.} \quad \frac{\binom{F}{0} \binom{N-F}{n} + \binom{F}{1} \binom{N-F}{n-1}}{\binom{N}{n}} \leq 1 - P$$

try  $n = 20$

$$\frac{\binom{90}{20} + 10 \binom{90}{19}}{\binom{100}{20}} = .42 \text{ which is } > \text{ than the acceptable "risk" of } 1-P=.20$$

. . 20 samples is too few

try  $n = 30$

$$\frac{\binom{90}{30} + 10 \times \binom{90}{29}}{\binom{100}{30}} = .194 \text{ which is within the acceptable "risk" of } .20$$

Therefore if 30 units are tested for 10,000 hours each and 1 random failure is observed, the MTTF of each of the original lot of 100 units is 100,000 hours with 80% confidence.

Date: 17 September 1964  
 Subject: HYPERGEOMETRIC DISTRIBUTION

EXPONENTIAL FAILURE RATE

W. Keyes  
 William Keyes

To approximate the failure rate of a lot of units, the exponential failure rate is assumed. Let  $F$  denote the failure rate of a unit, then the probability that a component will fail in a time  $t$  is given by  $1 - e^{-Ft}$ . The probability that a component will survive a time  $t$  is given by  $e^{-Ft}$ .

HYPERGEOMETRIC DISTRIBUTION

Given a lot of  $N$  units of which  $K$  are defective, the probability that a sample of  $n$  units will contain  $x$  defectives is given by the hypergeometric distribution. Let  $X$  = the number of defectives in a sample of  $n$  units. Then the probability that a sample of  $n$  units will contain  $x$  defectives is given by

$$P(X=x) = \frac{\binom{K}{x} \binom{N-K}{n-x}}{\binom{N}{n}}$$

where  $\binom{K}{x}$  is the number of ways of choosing  $x$  defectives from  $K$  defectives, and  $\binom{N-K}{n-x}$  is the number of ways of choosing  $n-x$  non-defectives from  $N-K$  non-defectives.

Thus, the probability that a sample of  $n$  units will contain  $x$  defectives is given by  $P(X=x)$ . The probability that a sample of  $n$  units will contain at least  $x$  defectives is given by  $P(X \geq x)$ .

$$P(X \geq x) = \sum_{i=x}^n \frac{\binom{K}{i} \binom{N-K}{n-i}}{\binom{N}{n}}$$